# Homework: Graph Algorithms

This document defines the **homework assignments** for the ["Algortihms" course @ Software University](https://softuni.bg/trainings/1194/Algorithms-September-2015). Please submit a single zip / rar / 7z archive holding the solutions (source code) of all below described problems.

## Distance between Vertices

We are given a **directed graph** consisting of N vertices and M edges. We are given also a set of **pairs of vertices**. Find the **shortest distance between each pair** of vertices or **-1** if there is no path connecting them. There are no specified requirements for the input and output, so you may hardcode the input and output values.

Examples:

|  |  |  |
| --- | --- | --- |
| **Input** | **Picture** | **Output** |
| Graph:  1 -> 2  2 ->  Distances to find:  1-2  2-1 |  | {1, 2} -> 1  {2, 1} -> -1 |
| Graph:  1 -> 4  2 -> 4  3 -> 4, 5  4 -> 6  5 -> 3, 7, 8  6 ->  7 -> 8  8 ->  Distances to find:  1-6  1-5  5-6  5-8 |  | {1, 6} -> 2  {1, 5} -> -1  {5, 6} -> 3  {5, 8} -> 1 |
| Graph:  11 -> 4  4 -> 12, 1  1 -> 12, 21, 7  7 -> 21  12 -> 4, 19  19 -> 1, 21  21 -> 14, 31  14 -> 14  31 ->  Distances to find:  11-7  11-21  21-4  19-14  1-4  1-11  31-21  11-14 |  | {11, 7} -> 3  {11, 21} -> 3  {21, 4} -> -1  {19, 14} -> 2  {1, 4} -> 2  {1, 11} -> -1  {31, 21} -> -1  {11, 14} -> 4 |

Hint: for each pair use **BFS** to find all paths from the source to the destination vertex.

## Areas in Matrix

We are given a matrix of letters of size N \* M. Two cells are neighbor if they share a common wall. Write a program to find the connected areas of neighbor cells holding the same letter. Display the total number of areas and the number of areas for each alphabetical letter.

Examples:

|  |  |  |
| --- | --- | --- |
| **Input** | **Picture** | **Output** |
| Number of rows: 6  aacccaac  baaaaccc  baabaccc  bbdaaccc  ccdccccc  ccdccccc |  | Areas: 8  Letter 'a' -> 2  Letter 'b' -> 2  Letter 'c' -> 3  Letter 'd' -> 1 |
| Number of rows: 3  aaa  aaa  aaa |  | Areas: 1  Letter 'a' -> 1 |
| Number of rows: 5  asssaadas  adsdasdad  sdsdadsas  sdasdsdsa  ssssasddd | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | | a | s | s | s | a | a | d | a | s | | a | d | s | d | a | s | d | a | d | | s | d | s | d | a | d | s | a | s | | s | d | a | s | d | s | d | s | a | | s | s | s | s | a | s | d | d | d | | Areas: 21  Letter 'a' -> 6  Letter 's' -> 8  Letter 'd' -> 7 |

**Hint**: Initially mark all cells as **unvisited**. Start a **recursive DFS traversal** (or BFS) from each unvisited cell and mark all reached cells as visited. Each DFS traversal will find one of the **connected areas**.

## Cycles in a Graph

Write a program to check whether an undirected graph is **acyclic** or holds any cycles.

|  |  |  |
| --- | --- | --- |
| **Input** | **Picture** | **Output** |
| C – G |  | Acyclic: Yes |
| A – F  F – D  D – A |  | Acyclic: No |
| E – Q  Q – P  P – B |  | Acyclic: Yes |
| K – J  J – N  N – L  N – M  M – I |  | Acyclic: Yes |
| K – X  X – Y  X – N  N – J  M – N  A – Z  B – P  I – F  A – Y  Y – L  M – I  F – P  Z – E  P – E |  | Acyclic: No |

**Hint**: Modify the Topological Sorting algorithm (source removal or DFS-based).

## Salaries

You can test your solution to the problem in the Judge system [here](https://judge.softuni.bg/Contests/Practice/Index/114#4).

We have a **hierarchy** between the employees in a company. Employees can have one or several direct managers. People who **manage nobody** are called **regular employees** and their salaries are **1**. People who manage at least one employee are called **managers**. Each manager takes a **salary** which is equal to the **sum of the salaries of their directly managed employees**. Managers cannot manage directly or indirectly (transitively) themselves. Some employees might have no manager (like the big boss). See a sample hierarchy in a company along with the salaries computed following the above described rule:



In the above example the employees 0 and 3 are regular employees and take salary 1. All others are managers and take the sum of the salaries of their directly managed employees. For example, manager 1 takes salary 3 + 2 + 1 = 6 (sum of the salaries of employees 2, 5 and 0). In the above example employees 4 and 1 have no manager.

If we have **N** employees, they will be indexed from 0 to N – 1. For each employee, you’ll be given a string with N symbols. The symbol at a given index **i**, either **'Y' or 'N'**, shows whether the current employee is a direct manager of employee **i**.

**Hint**: find the node with no parent and start a **DFS traversal** from it to calculate the salaries on the tree recursively.

### Input

* The input data should be read from the console.
* On the first line you’ll be given an integer N.
* On the next N lines you’ll be given strings with N symbols (either 'Y' or 'N').
* The input data will always be valid and in the format described. There is no need to check it explicitly.

### Output

* The output should be printed on the console. It should consist of one line.
* On the only output line print the sum of the salaries of all employees.

### Constraints

* N will be an integer in the range [1 … 50].
* For each i-th line, the i-th symbol will be 'N'.
* If employee A is the manager of employee B, B will not be a manager of A.
* Allowed working time for your program: 0.1 seconds. Allowed memory: 16 MB.

### Examples

|  |  |  |
| --- | --- | --- |
| **Input** | **Output** | **Comments** |
| 1  N | 1 | Only 1 employee with salary 1. |
| 4  NNYN  NNYN  NNNN  NYYN | 5 | We have 4 employees. 0, 1, and 3 are managers of 2. 3 is also a manager of 1. Therefore:  salary(2) = 1  salary(0) = salary(2) = 1  salary(1) = salary(2) = 1  salary(3) = salary(2) + salary(1) = 2 |
| 6  NNNNNN  YNYNNY  YNNNNY  NNNNNN  YNYNNN  YNNYNN | 17 |  |

## \* Break Cycles

You are given **undirected multi-graph**. Remove a minimal number of edges to **make the graph acyclic** (to break all cycles). As a result, print the number of edges removed and the removed edges. If several edges can be removed to break a certain cycle, remove the smallest of them in alphabetical order (smallest start vertex in alphabetical order and smallest end vertex in alphabetical order).

Examples:

|  |  |  |  |
| --- | --- | --- | --- |
| **Input** | **Picture** | **Output** | **Picture After Removal** |
| 1 -> 2, 5, 4  2 -> 1, 3  3 -> 2, 5  4 -> 1  5 -> 1, 3  6 -> 7, 8  7 -> 6, 8  8 -> 6, 7 |  | Edges to remove: 2  1 – 2  6 - 7 |  |
| K -> X, J  J -> K, N  N -> J, X, L, M  X -> K, N, Y  M -> N, I  Y -> X, L  L -> N, I, Y  I -> M, L  A -> Z, Z, Z  Z -> A, A, A  F -> E, B, P  E -> F, P  P -> B, F, E  B -> F, P |  | Edgeds to remove: 7  A - Z  A - Z  B - F  E - F  I - L  J - K  L - N |  |

##### Hints (Click on the arrow to show)

* Enumerate edges {**s**, **e**} in alphabetical order. For each edge {**s**, **e**} check whether it closes a cycle. If yes - remove it.
  + To check whether an edge {**s**, **e**} closes a cycle, temporarily remove the edge {s, e} and then try to find a path from **s** to **e** using DFS or BFS.